Origami in Engineering and Architecture

An art and science spanning Mathematics, Engineering and Architecture

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Objectives: get a glimpse of the underlying intricacies and fundamental mathematics involved in something as seemingly innocent as origami. Furthermore, get a feel for the engineering challenges when applying origami to engineering and architectural applications; in other words, when scaling up from paper models to the real world.

Origami Art

Origami is the name for the ancient Japanese art of paper folding. The word comes from Japanese, and is a combination of *'oru'*, which means 'fold' and *'kami'*, which means `paper'. For centuries origami has been practiced in the far East, and in the twentieth century it also gained popularity in the western world.

Although most people associate origami with simple models such as the classic origami crane, in the past twenty or so years origami folding has reached unprecedented heights. Increasingly complex and life-like art pieces are being created, which beggar belief that they can be folded from a single sheet of paper. Perhaps surprisingly, many of the recent advances in origami design are a result of the growing mathematical understanding of origami, and newly developed computational design tools. We shall briefly highlight some of the fundamental underlying mathematics in a later section, but first focus on a few recent developments in origami art.

A field of origami that has blossomed over the last few years is **origami tessellations**, where a fold pattern is tiled across the plane. Pioneered in the late eighties and early nineties, it has only recently entered mainstream folding. The models are becoming increasingly complex, and photo sharing websites such as Flickr have made the sharing and spreading of new ideas and techniques much faster. The origami tessellations also form part of a trend to move away from publishing step-by-step folding instructions, towards only publishing the crease patterns. This requires greater skill and knowledge of origami to interpret and fold, but increases the speed of publishing new patterns and also works for models whose folding cannot really be described in a linear fashion. Another area that is garnering attention is **curved folds**. Traditionally, origami folding makes use of straight fold lines, but curved folds open up new avenues of shapes and designs. Pioneering work was done in the 1970s by David Huffman and Ron Resch, but only recently their work has been rediscovered and fully appreciated.

Basic terminology

A *crease* is a line segment, or even a curve, on a piece of paper. Creases may be folded in one of two ways: as a *mountain fold*, forming a protruding ridge, or a *valley fold*, forming (not altogether surprisingly) a valley. A *crease pattern* is a collection of creases. A mountain-valley assignment is a specification of which creases should be folded as mountains, and which as valleys.

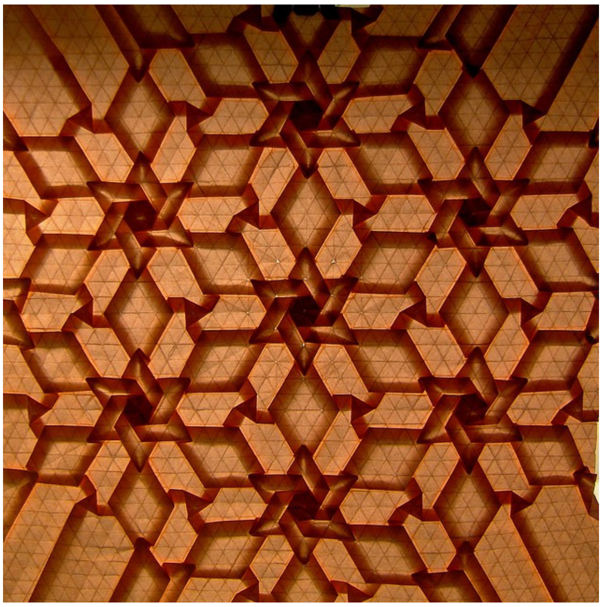


Figure 1: ‘Star Tessellation’ by Eric Gjerde.

Figure 2: ‘Rattle Snake’, Opus 539 by Robert Lang.

Figure 3: ‘untitled’ by David Huffman. Image courtesy of the Huffman family.

Origami Mathematics

Although it is certainly not the first thing that springs to mind when mentioning origami, it is actually deeply intertwined with **mathematics** and an increased understanding of the underlying mathematical laws has spurred the art of origami over the last two decades. Origami has an intrinsic geometry, that has attracted geometrists since the 19th century. They mostly focused on 2D geometrical constructions, and explored origami axioms to replace the standard techniques of a straight edge and compass. In recent years these axioms have been extended and formalized [4]. More interesting and useful for the origamists, however, are the developments in the understanding of the **foldability** of origami patterns.

Origami is traditionally folded from a flat sheet of paper. Mathematically this is represented as an infinitely thin sheet that cannot be stretched, and where only folding actions are allowed. This has a deep impact on the shapes that can be attained by origami folding. Differential geometry is the branch of mathematics that describes the local properties of curved surfaces. An important concept is **Gaussian curvature** [7], which is an intrinsic property of the surface. Crucially, the Gaussian curvature is invariant under bending operations. A flat piece of paper has zero Gaussian curvature at every point on it surface, which it will retain even when it is folded [8,11]. This strictly limits the geometries a flat sheet can assume! Although this may come across as a rather abstract concept, you are all familiar with this fact: for example, it is impossible to map a globe to a flat page in the atlas without distortion. Or conversely, you cannot wrap a piece of paper onto a sphere or a saddle. A flat sheet with zero Gaussian curvature is also referred to as a **developable surface**. A side effect of this is that in an origami crease pattern, all the angles at a fold vertex add up to 2π. This may seem obvious, but it plays an important role when trying to find a crease pattern that corresponds to a desired three-dimensional folded state.

Another interesting question in origami mathematics is the concept of **flat-foldability**. This means that in the folded state, when all creases are folded $\pm\pi$, the end result lies in a plane. For deployable structures (as discussed in the next section) this is of great interest, as it allows for minimal stowed dimensions. Flat foldability contains a number of interesting theorems and highlights a major challenge in origami mathematics: theorems are available for single vertices, but the extension to multiple vertices is (very) hard. The Kawasaki-Justin theorem states that

> *a single-vertex crease pattern defined by $\theta_1 + \theta_2 + \ldots + \theta_n = 2\pi$ is flat foldable if and only if n is even and the sum of the odd angles is equal to the sum of the even angles, or equivalently either sum is equal to π.*

and Maekawa's theorem adds that

> *in a flat-foldable single-vertex mountain-valley pattern defined by angles $\theta_1 + \theta_2 + \ldots + \theta_n = 2\pi$, the number of mountains M and the number of valleys V differ by ±2.*

These theorems are necessary, but not yet sufficient to prove the flat foldability of a single vertex; furthermore, they only cover *local flat-foldability.* Extending it to global flat foldability (taking into account all folds and vertices in a fold pattern) is proven to be NP hard (meaning it is computationally intractable). These theorems illustrate that there are simple, but deep, underlying rules to origami folding. However, describing global origami foldability remains hard!

Another important mathematical concept is **rigid foldability**, which focuses on the folding process, rather than the final folded state. The rigid origami paradigm requires that during the folding process the material only bends at the fold lines. An analogy would be rigid plates connected through hinges. Obviously, this assumption limits the patterns that can be folded. A surprising example is the classic paper grocery bag, which cannot fold flat without bending the facets [1]. In other words, the configuration space of the bag consists of two isolated points: the bag fully opened and fully closed. An interesting general result shows that a vertex must have at least four creases to be rigid foldable; in that case it will have one degree of freedom, and a closed-form solution for the fold angles can be found [8]. In recent years a great deal of progress has been made in understanding the necessary geometric conditions for rigid foldability, and has resulted in rigid foldable cylinders, as well as design methods for generalized quadrilateral mesh origami [13,14]. Tomohiro Tachi created a software program called Freeform Origami

http://www.tsg.ne.jp/TT/software/#ffo

which provides an interactive interface to modify existing, partly-folded, patterns by dragging around the vertices into a desired freeform surface; the resulting fold pattern is both rigid and flat foldable. The underlying mathematics and computational methodology of the software is described in [15].

The mathematical approach provides a far better understanding of origami folding, but also occasionally leads to surprises. For example the **hyperbolic paraboloid** has been around since the 1920s, but only recently it has been proven that it cannot actually mathematically be folded [3]! Instead, it relies on small areas of extensional deformation to assume the folded shape. In general, reality allows more flexibility than the mathematical formulations: stretching of material, bending of fold lines, shifting of hinge lines, all add to the possibilities of folding. Conversely, it also means that if you have a working paper model of an origami pattern, it does not mean you can convert it to an engineered version with panels and hinges! Fortunately, most conventional fold patterns are fine, and engineers therefore often make use of those.

One last comment about origami and mathematics, is to mention **mathematical origami design**. This does not concern itself with the fundamental theorems, but rather the ability to computationally create a crease pattern for a desired shape. The authority in this field is Robert Lang with his ‘Treemaker’ software, that uses a tree method to create origami bases [10]. It is this technology that truly spurred the development of ever more complex models.

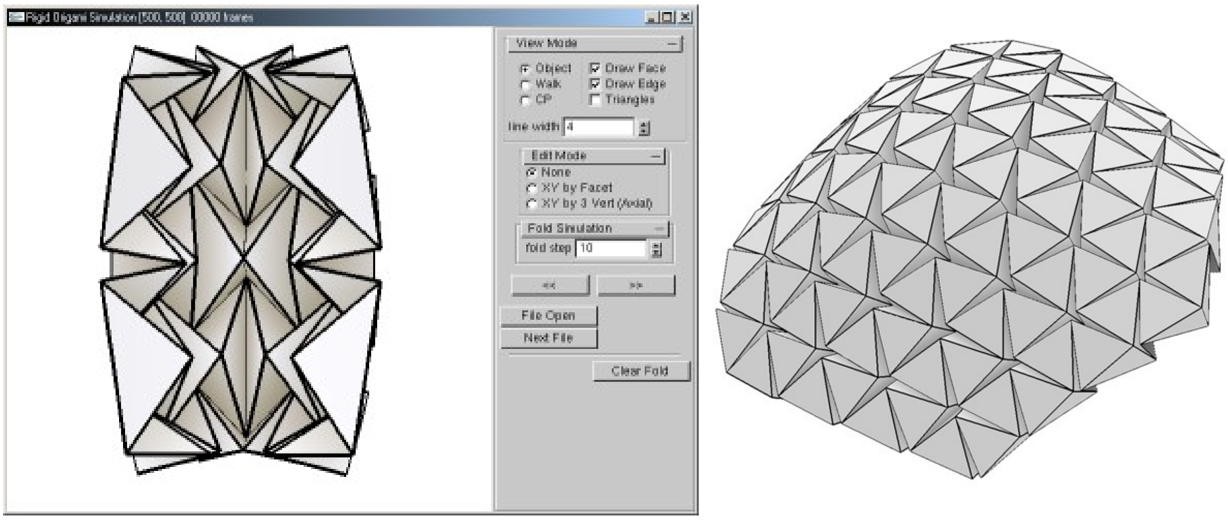


Figure 4: Tomohiro Tachi’s Rigid Origami simulator with a rendering of a folded pattern.

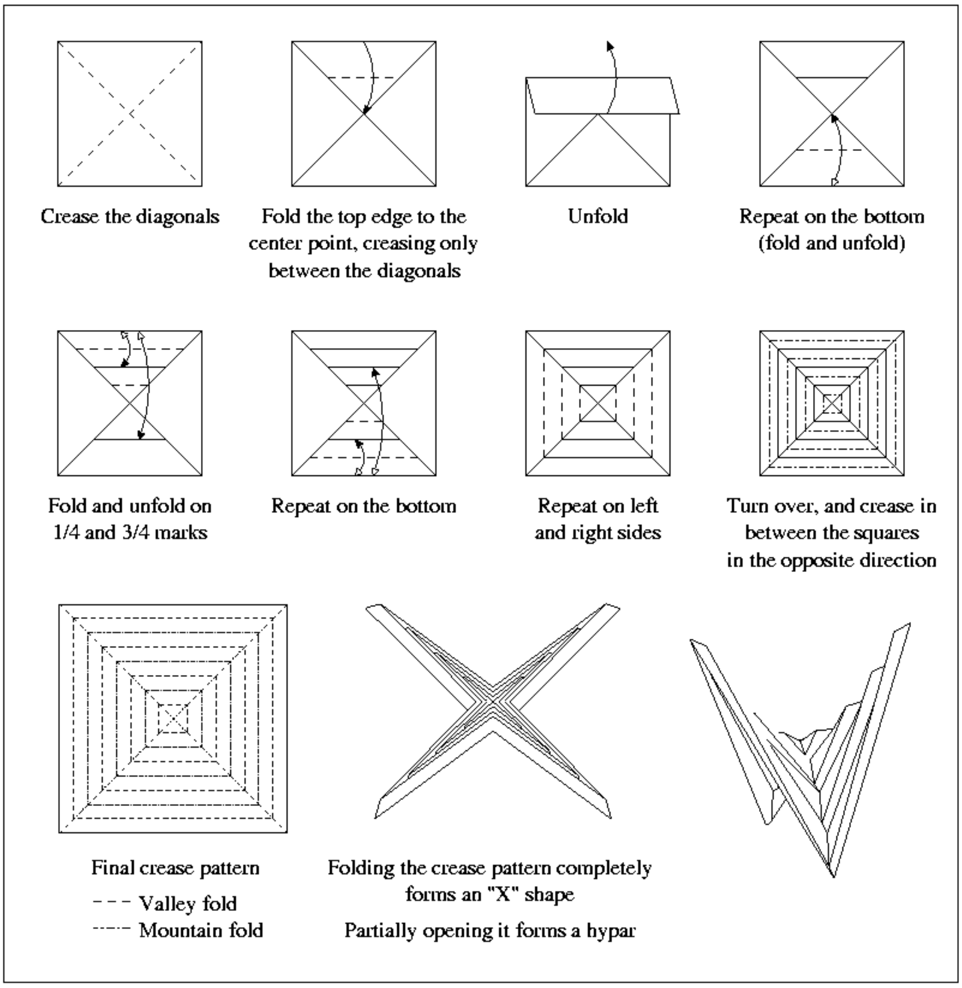


Figure 5: the 'unfoldable' pleated hypar (Hyperbolic Paraboloid). Image by Erik Demaine.

Origami Engineering

Engineers have also taken to origami as a source of inspiration for a variety of applications. Engineering applications of origami can broadly be categorized into three areas. The main application area is, probably unsurprisingly, **deployable structures.** These structures are designed to fold up compactly during stowage, and to be deployed on site. Applications range from space applications (*e.g.* solar panels, solar sails [5] and inflatable booms), to packaging, deployable shelters [16], and even medical stents [9]. Secondly, engineers make use of the ability of folding patterns to increase the **(bending) stiffness** of a sheet. This can for instance be used to create light-weight cores for sandwich panels [6]. The benefits over conventional honeycomb cores include the fact that the folded cores can be aerated, they can be designed with an intrinsic global curvature and can be continuously manufactured with minimal material deformations. Another application is folded plate roofs in architecture, where the folded plate design increases the bending stiffness, thus allowing to span greater spaces. A third field is **impact absorption**. Folded sheets can be used as packaging material, or as crashboxes in cars: the fold patterns induce higher buckling modes in the sacrificial crashboxes, thereby absorbing more energy and improving passenger safety.

Further applications are being developed. For example, the Advanced Structures Group at the engineering department is looking into using folded sheets as flexible surfaces for morphing structures [12]. It should be noted that most fold patterns used in engineering applications tend to be comparatively simple, largely due to difficulties in manufacturing.

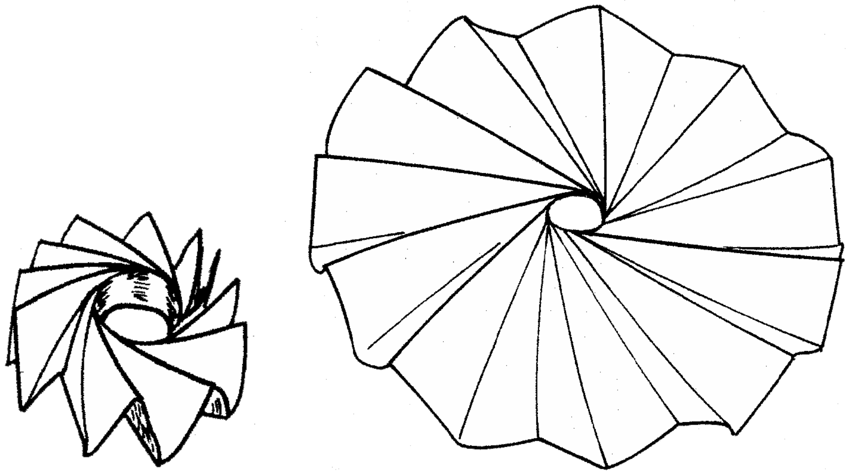


Figure 6: wrapping pattern for a solar sail [5].

Figure 7: deployable origami stent, made from shape memory alloy [9].

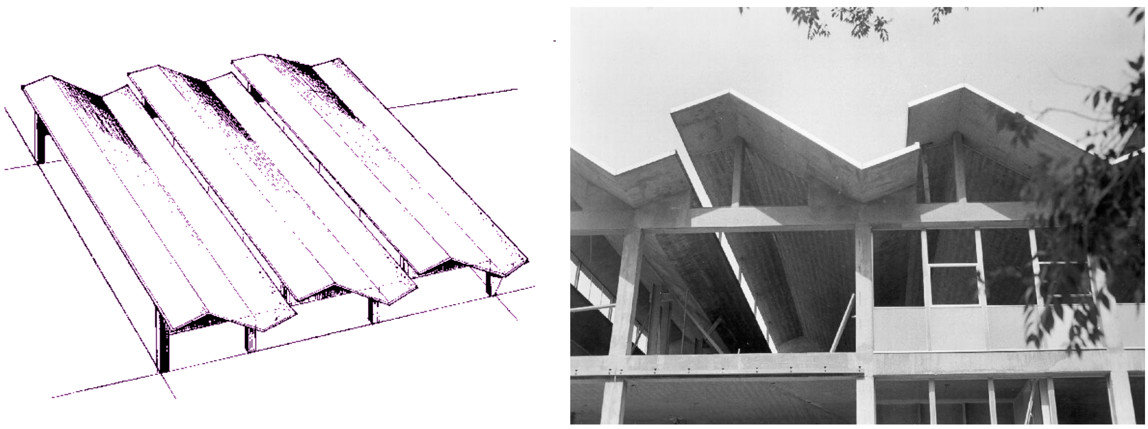


Figure 8: simple folded plate roof, providing increased bending stiffness.

Origami Engineering : (mechanical) modeling methods

A brief note is in order regarding the modeling methods used when applying origami to engineering applications. The choice of method depends largely on the application, but also on the intrinsic properties of the fold pattern: describing the kinematics of a rigid foldable pattern will require a different approach, than, say, investigating the impact absorption qualities of an origami crashbox.

For example, rigid origami can be modeled in a variety of manners. It can for instance be modeled by considering constraints on every vertex [15], by modeling the fold pattern as a pin-jointed bar framework with inextensional bars [12] or using mechanism theory [1]. However, once the mechanism has reached its desired folded state, one may be interested in its structural (rather than kinematic) properties. The material stiffness becomes important, and the structural modelling can be continued using using Finite Element Analysis. There also exist geometric modelers to recreate the crumpling of paper, or the geometry of curved folds. Essentially, the modeling method depends on the application: do you need a purely kinematic method, a stiffness method, something in between? The answers depend on the questions you ask!

Origami Architecture

Finally, we arrive at the applications of origami in architecture. An obviously important aspect is the **visual appeal** of origami folding patterns. Those visual patterns, however, can also be united with improved mechanical properties: as indicated previously, their increased bending stiffness means **folded plate roofs** can span larger areas. Furthermore, folded plate roofs can provide a means to approximate curved surfaces. As discussed in the mathematics section, flat plates cannot be bent into doubly curved shapes without stretching of the material. However, the globally curved shape can be approximated using flat panels, or at least piece-wise developable sheets. Another reason for looking into origami for architectural applications can come from the choice of building material. Timber panels are a renewable building material and will potentially be used more often in the future. Research is ongoing to use origami as source of inspiration to use this material effectively [2]. Lastly, Origami can be used to create deployable structures and kinematic façades and roofs. This area is currently very much under-explored, and some of the technical challenges will form part of the workshop.

Figure 10: origami-inspired timber chapel, designed by H. Buri and Y. Weinand.

Workshop

A workshop will follow immediately after the lecture. In it you will explore some of the challenges in origami engineering and architecture. The group assignments are purposely open-ended and will let you brainstorm about technical requirements, feasibility, materialization, and applications. The results will be presented to the other groups at the end of the afternoon. This can be with a short presentation, a poster or some scale models. We encourage you to bring along a laptop, as well as craft tools such as a Stanley knife, a pair of scissors, tape, blu-tack, paperclips, ruler, *etc*.

Assignment: Folding Patterns

In preparation for the lecture and workshop you are asked to fold **three** origami patterns. Two compulsory patterns are provided at the back of this handout, and a third of choice can be downloaded from my website:

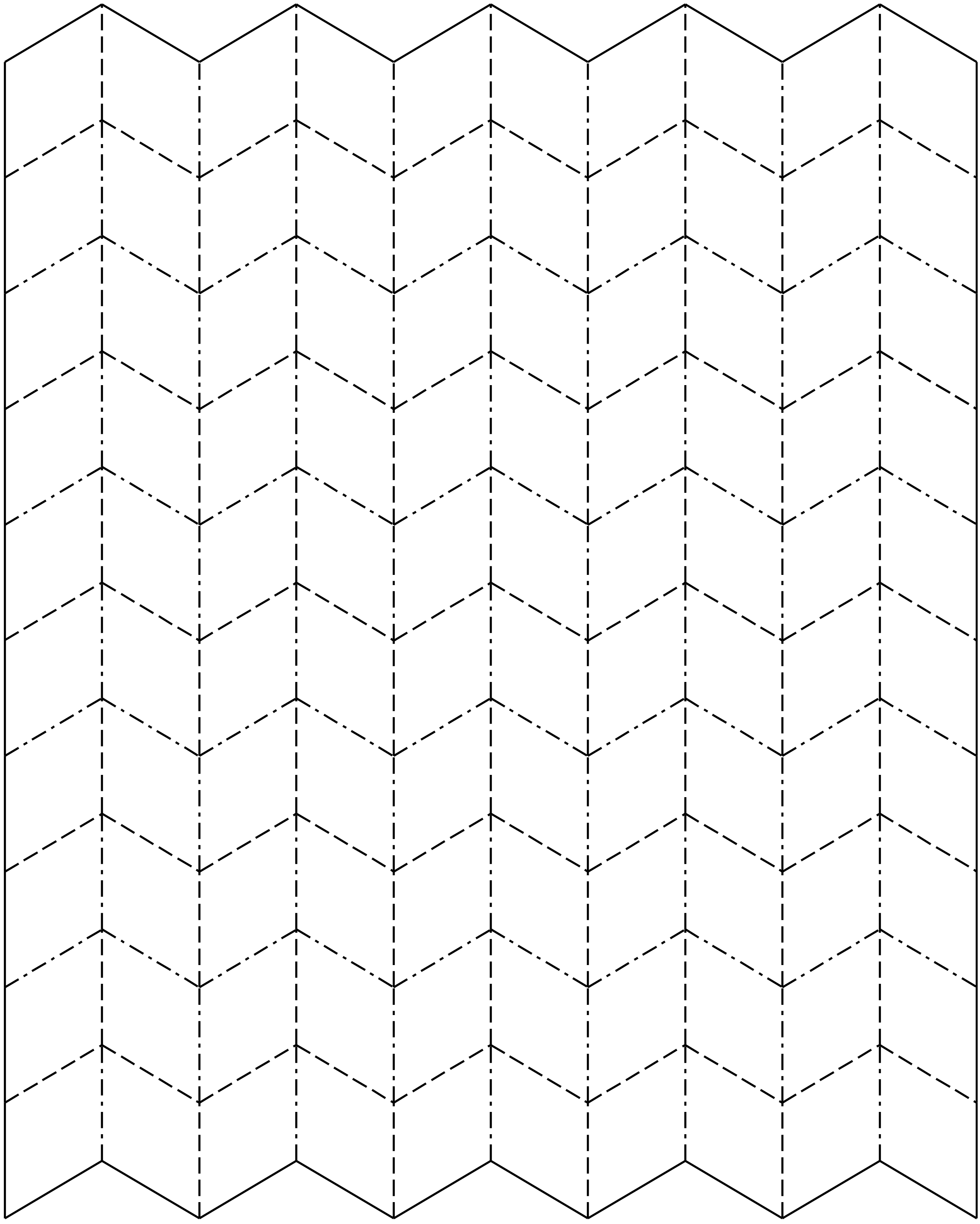
http://www2.eng.cam.ac.uk/~ms652/teaching/archeng2012.html

They will give you a feel for the difficulties of manufacturing these types of folded structures and will improve your folding dexterity when/if you need to make a model during the workshop. It's also fun to do! Once you've folded them, play around to see what shapes they can attain and think how you might be able to use or modify them.

Instructions: valley folds are indicated as - - - - - -, whereas mountain folds are - · - · - · - . The best method is to score and crease all the fold lines individually before trying to collapse the overall structure. The more accurately you do it, the nicer the final result will be. *Please do not underestimate the time needed to fold these patterns! I expect inexperienced folders to need about half an hour for each pattern.*

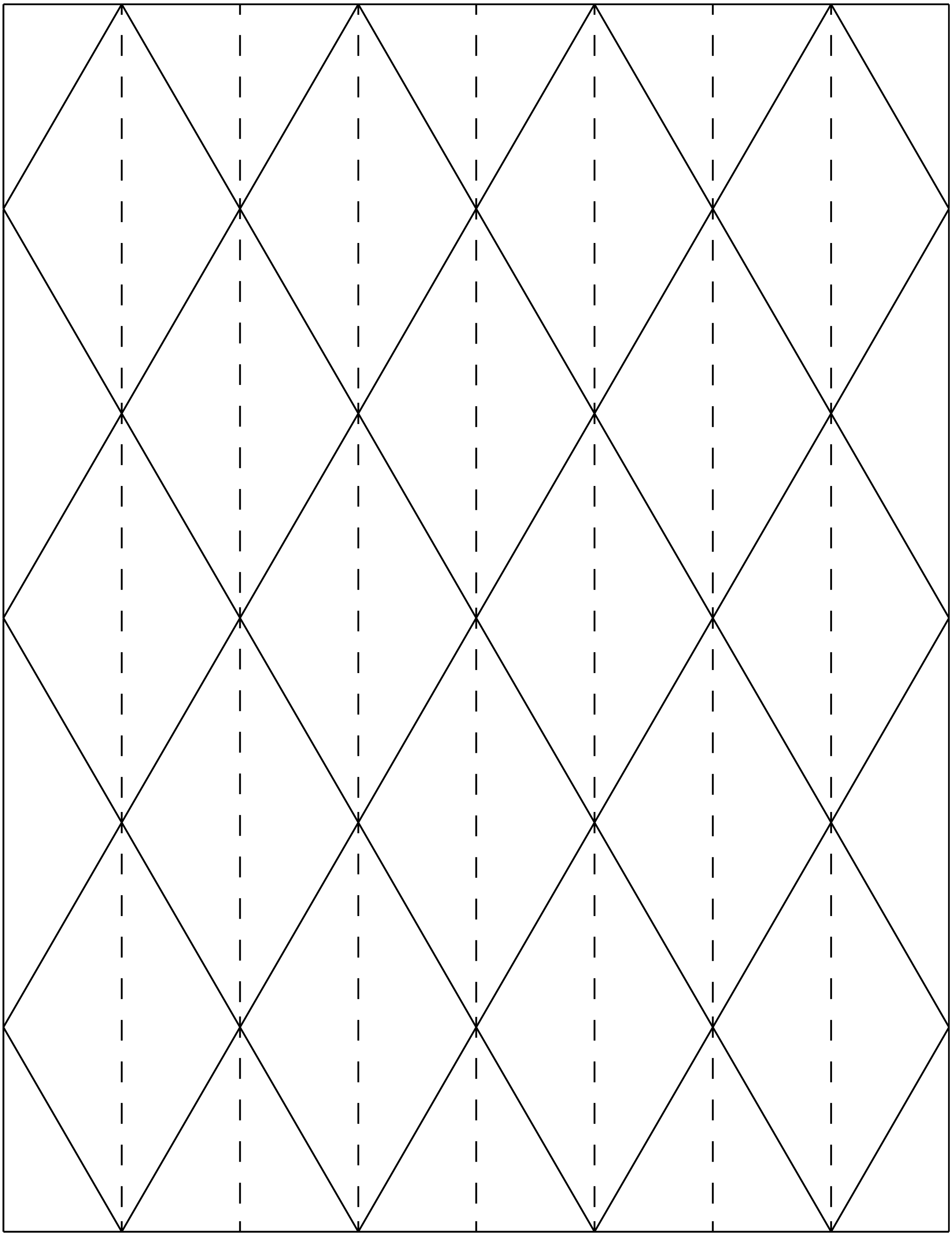
Miura-ori

The *Miura-ori* pattern is the classic (and simplest) technical folding pattern, and has been used from deployable solar panels to light-weight cores for sandwich panels. It goes by a multitude of names but is best known among engineers as the *Miura-ori* pattern, named after the Japanese engineer Koryo Miura who made it famous. The geometry can easily be adapted to create globally curved surfaces [2].



Yoshimura

This too is a classic pattern engineering folding pattern. It was initially inspired by the local post-buckling patterns in thin-walled cylinders, and named after the scientist who described them. It is commonly used by people exploring the design of collapsible shelters using Origami techniques [16,17] and has been proposed for inflatable booms in space structures.



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